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A Finite Element Modeling of the Impact of Internal Erosion on the Stability of a Dike

Jie Yang¹; Zhen-Yu Yin²; Pierre-Yves Hicher³; and Farid Laouafa⁴

¹LUNAM Univ., Ecole Centrale de Nantes, GeM UMR CNRS 6183, Nantes, France; INERIS, Verneuil en Halatte, France. E-mail: jie.yang@ec-nantes.fr

²LUNAM Univ., Ecole Centrale de Nantes, GeM UMR CNRS 6183, Nantes, France. E-mail: zhenyu.yin@ec-nantes.fr

³LUNAM Univ., Ecole Centrale de Nantes, GeM UMR CNRS 6183, Nantes, France. E-mail: pierre-yves.hicher@ec-nantes.fr

⁴INERIS, Verneuil en Halatte, France. E-mail: farid.laouafa@ineris.fr

Abstract

This paper presents a numerical model of internal erosion based on the approach of continuous porous media. The soil skeleton saturated by a pore fluid is treated as the superposition of four constituents in interaction: solid skeleton, erodible fines, fluidized particles, and fluid. The detachment and transport of fine particles are modeled by the mass exchange between solid and fluid phases. In order to take into account the influence of the change of void ratio induced by internal erosion on the soil skeleton, a critical state based constitutive model is used to calculate the effective stress-strain response of the soil skeleton. This coupled hydro-mechanical analysis is applied to study the impact of erosion on a dike. The numerical simulations show the progressive development of internal erosion within the foundation of the dyke. The effect of the mechanical degradation due to internal erosion is demonstrated by evaluating the factor of safety of the dyke slope by means of the shear strength reduction method.

INTRODUCTION

Internal erosion is a phenomenon of progressive degradation of soil induced by hydrodynamic forces, reducing progressively its mechanical properties, and eventually leading to failure. This phenomenon has been studied by laboratory testing and numerical modeling (Reddi et al. 2000; Sterpi 2003; Cividini et al. 2004; Wan et al. 2004; Cividini et al. 2009; Fujisawa et al. 2010). However, most of the numerical studies of coupled hydro-mechanical analysis for internal erosion have used a linear elastic constitutive model, whereas a more realistic elastoplastic constitutive model should be considered in order to discuss the stability of earth structures during internal erosion.

This paper presents a coupled hydro-mechanical analysis for internal erosion using the approach of continuous porous media (Vardoulakis et al. 1996; Stavropoulou et al. 1998). The soil skeleton saturated by a pore fluid is treated as the superposition of four constituents in interaction. The internal erosion of the soil skeleton and the transport of fine particles by pore water are modeled by the mass exchange between solid and fluid phases. The stress-strain relationship for the solid skeleton is calculated by a critical state based constitutive model (Yin et al. 2013) which can take into account the progressive degradation of the soil mechanical properties induced by internal erosion. The balance equations of the material system are implemented into a finite element code. The discretized equations are linearized and iteratively solved by the Newton-Raphson method to ensure the equilibrium at the current configuration. The finite element code is then applied for evaluating the influence of internal erosion on a dike slope stability by calculating the safety factor by using the shear strength reduction method.

DEFINITION OF THE HYDRO-MECHANICAL EQUATIONS

Mass balance and particle transport

The mass balance equations are based on the porous media theory (Boer 2000; Schrefler 2002). The saturated porous medium is modeled as a material system consisting of the stable fabric of the solid skeleton (s_s), erodible fines (f_s), fluidized particles (f_c) and fluid (f_f). In current conditions, fines can either behave as a fluid-like (described as fluidized particles f_c) or a solid-like (described as erodible fines f_s) material. Thus a liquid-solid phase transition process is taken into account in the present model by a mass/volume production term in the corresponding partial mass/volume balances of f_s and f_c .

In a given Representative Elementary Volume (REV) with the volume dV consisting of the four constituents, the volume fraction of a single constituent α is expressed as:

$$n^\alpha = \frac{dV^\alpha}{dV} \quad (1)$$

with $\alpha = \{s_s, f_s, f_c, f_f\}$.

The local form of the partial balance of mass for the α phase is given as (Schaufli et al. 2013):

$$\partial_t(\rho^\alpha) + \text{div}(\rho^\alpha \mathbf{v}^\alpha) = \hat{\rho}^\alpha \quad (2)$$

Notice that in this study the hydro-mechanical dispersion is disregarded. The partial density ρ^α is defined as the ratio between the mass dm^α of the phase α with respect to the total volume dV of the REV, which leads to a relation between partial densities ρ^α and the effective densities $\rho^{\alpha R}$:

$$\rho^\alpha = \frac{dm^\alpha}{dV} = \frac{dm^\alpha}{dV^\alpha} \frac{dV^\alpha}{dV} = \rho^{\alpha R} n^\alpha \quad (3)$$

The constraint for the mixture mass production $\sum_{\alpha} \hat{\rho}^{\alpha} = 0$ in the system guarantees that there is no mass production in the total mixture of the assembly. The partial mass balance is then reduced to a partial volume balance:

$$\partial_t (n^{\alpha}) + \text{div}(n^{\alpha} \mathbf{v}^{\alpha}) = \hat{n}^{\alpha} \quad (4)$$

where \hat{n}^{α} is the volumetric mass exchange term which will be discussed in the following part.

Moreover, it is assumed that fluid and fluidized particles have at any time the same velocity. The solid skeleton is assumed to be deformable and non-erodible. The porosity field ϕ , the amount of erodible fines f_s , the concentration of the fluidized particles f_c are defined as follows:

$$\begin{aligned} \phi &= \frac{dV_v}{dV} = \frac{dV^{f_f} + dV^{f_c}}{dV} = n^{f_f} + n^{f_c} \\ f_s &= \frac{n^{f_s}}{n^{s_s} + n^{f_s}} \\ f_c &= \frac{n^{f_c}}{\phi} \end{aligned} \quad (5)$$

Considering the transformation of the state of fines from part of the solid skeleton to fluidized particles $\hat{n}^{f_c} = -\hat{n}^{f_s}$, $\hat{n}^f = 0$, the mass balance equations are given by the following expressions:

$$\partial_t \phi = -\hat{n}^{f_s} \quad (6)$$

$$\partial_t (c\phi) + \text{div}(c\mathbf{q}) = \hat{n}^{f_c} \quad (7)$$

$$\text{div}(\mathbf{q}) + \partial_t (\varepsilon_v) = 0 \quad (8)$$

Constitutive equation for mass exchange

Equations (6)-(8) are not sufficient for solving a boundary value problem since a constitutive relation for the mass exchange term is needed. A constitutive law for the rate of eroded mass, suggesting that erosion is driven by the discharge of the fluidized particles $f_c |\mathbf{q}|$, is given by the following relation (Vardoulakis et al. 1996; Uzuoka et al. 2012):

$$\hat{n}^{f_s} = \lambda_e f_c |\mathbf{q}| \quad (9)$$

where the coefficient λ_e has the inverse length dimension and must be determined experimentally.

Darcy's law

In this study, the flow in the porous medium is considered to be governed by Darcy's law which states that the flow rate is driven by the gradient of the pore fluid pressure:

$$\mathbf{q} = -\frac{k}{\eta_k \bar{\rho}} \mathbf{grad}(p) \quad (10)$$

where η_k is the kinematic viscosity of the fluid, p is the pore fluid pressure and $\bar{\rho}$ is the partial density of the mixture defined as:

$$\bar{\rho} = f_c \rho_s + (1 - f_c) \rho_f \quad (11)$$

with ρ_s the density of solid and ρ_f the density of fluid. The permeability k of the porous medium depends on the porosity ϕ . In this study, the permeability k is defined as (Stavropoulou et al. 1998):

$$k = k_0 \frac{\phi^{k_1}}{(1-\phi)^{k_2}} \quad (12)$$

Constitutive equations for the solid phase

The mechanical behavior of the solid skeleton is reproduced by a critical state based constitutive model (Yin et al. 2013) in order to take into account the hydro-mechanical coupling effect induced by internal erosion.

The total strain rate is decomposed into elastic and plastic parts:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p \quad (13)$$

The elastic behavior is assumed to be isotropic with the bulk modulus K having the form proposed by Richart et al. (1970):

$$\dot{\epsilon}_{ij}^e = \frac{1+\nu}{3K(1-2\nu)} \sigma'_{ij} - \frac{\nu}{3K(1-2\nu)} \sigma'_{kk} \delta_{ij} \quad (14)$$

$$K = K_0 \cdot p_{at} \frac{(2.97 - e)^2}{1 + e} \left(\frac{p'}{p_{at}} \right)^\zeta \quad (15)$$

where K_0 and ζ are elastic parameters; ν is Poisson's ratio; p' is the mean effective stress; p_{at} is the atmospheric pressure used as reference pressure ($p_{at} = 101.325$ kPa).

The plastic strain is based on the flow rule:

$$\dot{\epsilon}_{ij}^p = d\lambda \frac{\partial g}{\partial \sigma'_{ij}} \quad (16)$$

with the yield surface for shear sliding expressed as:

$$f = \frac{q}{p'} - \frac{M_p \epsilon_d^p}{k_p + \epsilon_d^p} \quad (17)$$

where f is the yield function; q is the deviatoric stress; k_p relates to the plastic shear modulus; M_p is the stress ratio corresponding to the mobilized peak strength determined by the mobilized peak friction angle ϕ_p ($M_p = 6 \sin(\phi_p) / (3 - \sin(\phi_p))$ in compression); ϵ_d^p is the deviatoric plastic strain.

The potential surface including the stress-dilatancy expression is written as follows:

$$\begin{aligned} \frac{\partial g}{\partial \sigma'_{ij}} &= \frac{\partial g}{\partial p'} \frac{\partial p'}{\partial \sigma'_{ij}} + \frac{\partial g}{\partial s_{ij}} \frac{\partial s_{ij}}{\partial \sigma'_{ij}} \text{ with} \\ \frac{\partial g}{\partial p'} &= D \left(M_{pt} - \frac{q}{p'} \right); \quad \frac{\partial g}{\partial s_{ij}} = \sqrt{\frac{3}{2}} \frac{s_{ij}}{\sqrt{s_{ij} s_{ij}}} \end{aligned} \quad (18)$$

where $s_{ij} = \sigma'_{ij} - p' \delta_{ij}$; D is the stress-dilatancy parameter; $M_{pt} = 6 \sin(\phi_{pt}) / (3 - \sin(\phi_{pt}))$ can be calculated from the mobilized phase transformation friction angle ϕ_{pt} .

The mobilized peak friction angle and the phase transformation friction angle are linked to the critical friction angle ϕ_μ (corresponding to the critical state line, CSL, in the p' - q plane), and the critical void ratio e_c (obtained from the critical state line, CSL, in the e - $\log p'$ plane) as follows:

$$\tan \phi_p = \frac{e_c}{e} \tan \phi_\mu; \quad \tan \phi_{pt} = \left(\frac{e_c}{e} \right)^{-1} \tan \phi_\mu \quad (19)$$

$$e_c = e_{ref} - \lambda \ln \frac{p'}{p_{ref}} \quad (20)$$

where e_{ref} is the reference critical void ratio corresponding to $p_{ref} = p_{at}$; λ is the slope of the CSL in the e - $\log p'$ plane.

The plastic multiplier, $d\lambda$, can be calculated in a conventional way according to plasticity theory:

$$d\lambda = \frac{\left[\frac{\partial f}{\partial \sigma_{ij}} \right]^T}{\left[\frac{\partial f}{\partial \sigma_{ij}} \right]^T \left[D_{ijkl} \frac{\partial g}{\partial \sigma_{kl}} \right] - \frac{\partial f}{\partial \varepsilon_d^p} \frac{\partial g}{\partial q}} \quad (21)$$

Combining Equations (13) to (21), the stress-strain relationship can be obtained and is expressed in terms of effective stresses applied on the solid skeleton.

Hydromechanical coupling effect

Conceptually, the hydro-mechanical coupling effect can be divided into two parts. In the first part, the internal erosion results in an evolution of the material porosity, which leads to the change of permeability controlled by Eq.(12). As a consequence, the pore pressure and the effective stresses will be redistributed. In the second part, the loss of fine particles due to seepage flow affects the mechanical behavior of the soil.

SIMULATION OF THE IMPACT OF INTERNAL EROSION ON A DIKE

To illustrate the suggested model, the stability of a dike slope was analyzed. In this example, a karst collapse underneath the foundation led to a cavity through which the underground water could flow out. Erosion takes place due to local hydraulic gradients in the vicinity of the cavity. The computations are aimed at evaluating the impact of internal erosion on the dike slope stability. The geometry and dimension of this numerical example are schematically shown in Figure 1 according to the typical cross section of the dike of Val d'Orléans (Alboresha 2016). The water level is at the top of the alluvium which is fully saturated whereas the dike is considered dry. The problem is analyzed under the assumption of plane strain condition. The material parameters are summarized in Table 1.

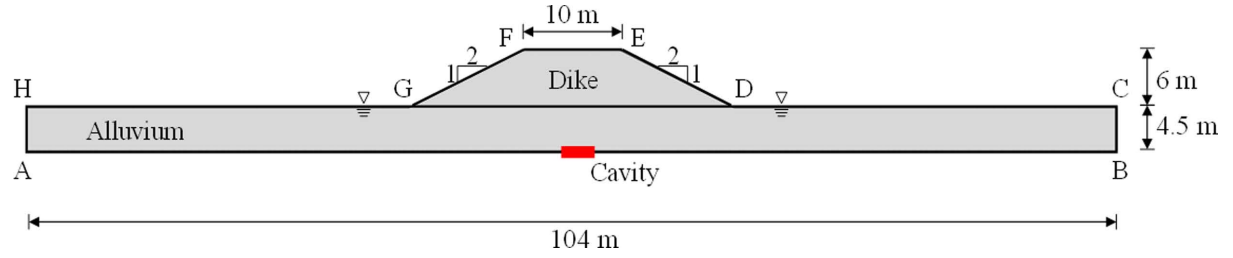


Figure 1. Schematic configuration of the dike.

Table 1. Material parameters

Physical and mechanical properties			Hydraulic properties		
Density of fluid	ρ_f	1.0 g/cm ³	Kinematic viscosity of fluid	η_k	5.0E-6 m ² s ⁻¹
Density of solids	ρ_s	2.65	Intrinsic permeability	k_0	5E-9 m ²
Bulk modulus	K_0	130 p_{at}	Erosion coefficient	λ_e	1E-3 m ⁻¹
Shear modulus	G_0	78 p_{at}	Initial concentration	$f_{c,0}$	0.01
Critical void ratio of p_{ref}	e_{ref}	0.936	Initial erodible fines in soil skeleton	$f_{s,0}$	0.25
CSL parameter	λ	0.0936	Initial void ratio	ϕ_0	0.3939
CSL parameter	κ	0.365	Permeability parameter	k_1	3
Critical friction angle	ϕ_u	31.6°	Permeability parameter	k_2	2

Two representative results are presented in Figures 2 and 3. Figure 2 shows the evolution of the porosity. As internal erosion takes place, the porosity which is initially uniform (see Figure 3a) increases with time (see Figure 3b-3c). The increase is particularly critical in the vicinity of the cavity, which leads to a highly eroded zone. As the dike is dry, the erosion process takes place only in the alluvium.

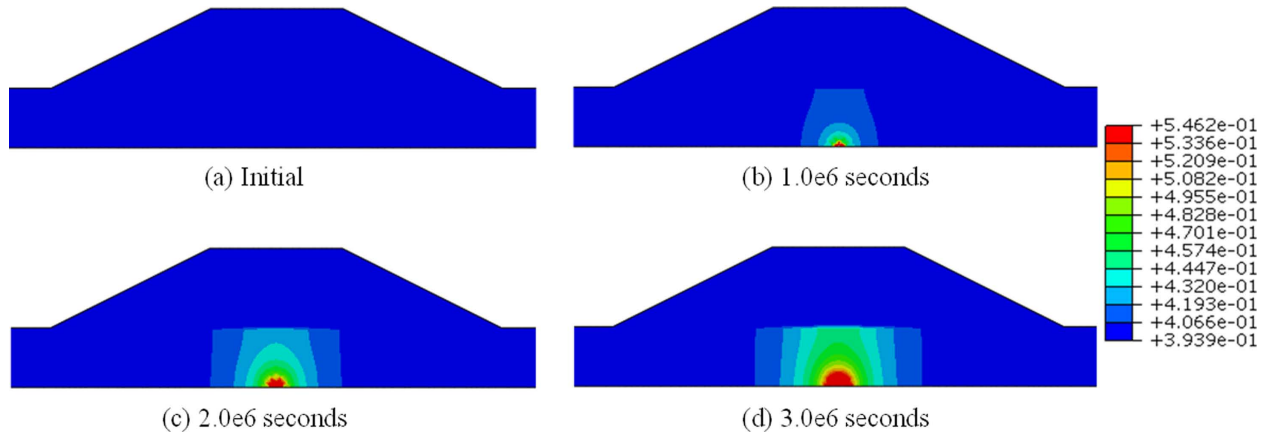


Figure 2. Spatial distribution of porosity with time.

The impact of internal erosion on the dike slope stability is evaluated by analyzing the evolution of the factor of safety F_s by using the shear strength reduction method. Since F_s is defined as the ratio between the actual soil shear strength and the minimum shear strength required to prevent failure, the simulations are run for a series of trial factors of safety F^{trial} with ϕ adjusted according to the equation:

$$\phi^{trail} = \arctan\left(\frac{1}{F^{trail}} \tan \phi\right) \quad (22)$$

Figure 3 shows the vertical displacement at point E obtained when the shear strength is progressively reduced in small steps, starting from a value of $F^{trail} = 0.65$. It can be seen that with the amount of erosion increasing with time, the degradation of the soil develops, the factor of safety F_s decreases, which indicates that the risk of mechanical instability increases.

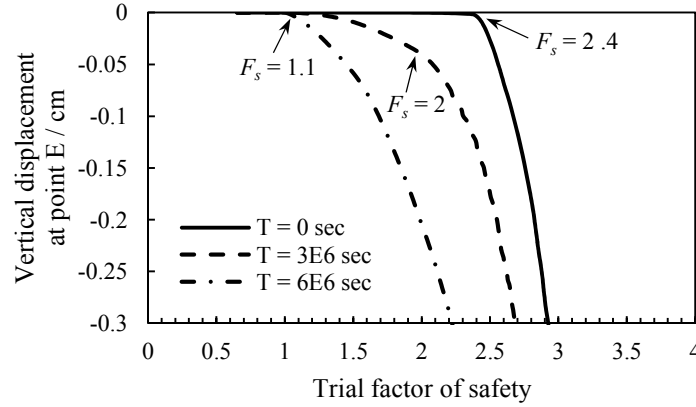


Figure 3. Vertical displacement as the trial factor of safety is increased for cases with different erosion time.

CONCLUSION

In this study, a numerical approach for modeling the internal erosion of soils is presented. The governing differential equations were formulated based on the mass balance of four constituents and then implemented into a finite element code. The internal erosion of the soil skeleton and the transport of fine particles in the fluid were modeled by the mass exchange between the solid skeleton and the pore water. Furthermore, a critical state based constitutive model was used to take into account the hydro-mechanical coupling effect induced by internal erosion. The example of a dyke was used to demonstrate the applicability of the model to simulate the evolution of internal erosion with time. The effects of internal erosion led to an increase of the soil porosity and to the degradation of the mechanical properties of the dyke foundation in time and space. The shear strength reduction method was adopted to evaluate the stability of the dike slope. The results show that the safety factor was reduced by the development of internal erosion, increasing the risk of mechanical instability.

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